Magnetic Field Modeling of Halbach Permanent Magnet Array

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Abstract

In this paper the magnetic flux density distribution of a Halbach permanent magnet array for magnetically levitated planar motor is modelled analytically. The analytical first order harmonic model is derived initially from Fourier harmonic waves of magnetic field. Then the real magnetic field of a Halbach permanent magnet array is measured on a Teslameter-centred measurement platform and the measured results are analysed. Next, the key parameter of analytical model is fitted and the analytical model and the measured real magnetic field are compared. The analytical and compared results indicate that the analytical model accords with the measured real magnetic field, in addition, the error root of mean square between the model value and measured value of sinusoidal magnetic flux density distribution is less than 43.554 Gauss, and the relative error to the amplitude of sinusoidal magnetic flux density distributin is less than 5%. The constructed first order harmonic model represents analytically the magnetic flux density distribution of Halbach permanent magnet array and it will facilitate the modelling of electromagnetic force/torqe and the commutation of the magnetically levitated planar motor.

Keywords

Halbach Permanent Magnet Array; The First Order Harmonic Model; Magnetic Flux Density Distribution; Magnetic Field

Introduction

Magnetically levitated stages (MLS) have been developed over the last decade, as alternatives to air bearing stages constructed of stacked linear motors. This is because MLS with the capability to obtain six degrees-of freedom motion by means of its single mover, can be operated in vacuum environment, for example in extreme-ultraviolet lithography or nanoimprint lithography equipments. This kind of stage involving in two parts, stator and mover, can be constructed in two ways. One way is that the mover consists of coil array and the stator of permanent magnet array (MLS with moving coils). The other way is that the mover consists of permanent magnet array and the stator of coil array (MLS with moving magnets). In recent years, a novel permanent magnet array named Halbach has frequently been used to construct MLS. This Halbach permanent magnet array (HPMA) can increase and concentrate the magnetic flux density near the coils compared with those of ordinary NS-array.

Halbach produced the first publication concerning the principle of HPMA. Trumper and Compter analyzed and computed the force generation above a HPMA. Yet they have not modeled the magnetic field of HPMA individually. Zhu and Howe gave an overview of HPMA, with the main objective of enhancing the magnetic field performance. Jansen et al. who built the surface charge model and the harmonic model of the magnetic field of HPMA finally obtained an analytical model used to model the electromagnetic force/torque and commutate their MLS.

In this paper, the analytical magnetic field model, named magnetic flux density distribution of HPMA, is established initially according to the way Jansen et al. proposed. Then the real magnetic field measured on a Teslameter-centred measurement platform is observed and analyzed. Afterward, the key parameter of the analytical magnetic field model is fitted and the error between the analytical model value and the measured data is analyzed. Last, the study is concluded.

Modeling of Magnetic Field of HPMA

As shown in Fig.1, the HPMA consists of many permanent magnets which can be classed into two types, big magnets and small magnets. They are pasted on a base board according to specific rules. In Fig. 1, the arrow denoting the magnetization direction of the permanent magnet points from the south pole to north pole of the small megnets. "∘" and "×" mean the north and south pole of the big megnets, respectively. On the surface of the array, two coordinate systems are defined, the initial coordinate system ${}^{m}o\text{-}{}^{m}x{}^{m}y{}^{m}z$

and the fixed coordinate system O-XYZ. The coordinate system O-XYZ is obtained by rotating the coordinate system ${}^mo$-${}^mx{}^my{}^mz$ clockwise 45°. τ and τ_n are magnetic pole pitch in two coordinate systems and $\tau=\sqrt{2}\tau_n$.

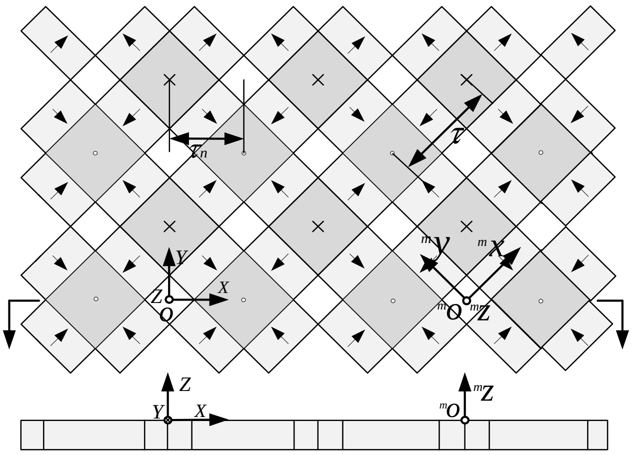

FIG. 1 MODELING OF MAGNETIC FIELD OF HPMA

In the light of the way Jansen proposed, as the neglection of all the higher order harmonic of Fourier harmonic waves, the 1st order harmonic model of magnetic field in ${}^mo$-${}^mx{}^my{}^mz$ coordinate system, can be described as:

$$B_m({}^mx,{}^my,{}^mz)=[B_{mx},B_{my},B_{mz}]^{\mathrm{T}}$$

$$=-\exp\left(-\frac{\sqrt{2}\pi}{\tau}{}^mz\right)\begin{bmatrix} B_{xy}\cos(\frac{\pi}{\tau}{}^mx)\sin(\frac{\pi}{\tau}{}^my) \\ B_{xy}\sin(\frac{\pi}{\tau}{}^mx)\cos(\frac{\pi}{\tau}{}^my) \\ \sqrt{2}B_{xy}\sin(\frac{\pi}{\tau}{}^mx)\sin(\frac{\pi}{\tau}{}^my) \end{bmatrix}, \quad (1)$$

Where B_{xy} is the amplitude of the 1st order harmonic of the magnetic flux density components at Z = 0. For HPMA whose size is defined, B_{xy} is a constant. So it is the key parameter of the analytical model of the magnetic field.

Then, by rotation coordinate transformation, the model in O-XYZ coordinate system turns to be:

$$B_m(X,Y,Z)=[B_{mX},B_{mY},B_{mZ}]^{\mathrm{T}}$$

$$=\frac{B_{xy}}{\sqrt{2}}\exp\left(-\frac{\pi}{\tau_n}Z\right)\begin{bmatrix} -\sin(\frac{\pi}{\tau_n}X) \\ \sin(\frac{\pi}{\tau_n}Y) \\ \cos(\frac{\pi}{\tau_n}X)-\cos(\frac{\pi}{\tau_n}Y) \end{bmatrix}. \quad (2)$$

Equations (2) is the analytical model of magnetic field of HPMA, i.e. the magnetic flux density distribution (MFDD). B_{mX}, B_{mY}, and B_{mZ} are the X, Y, and Z components, respectively.

Magnetic Field Measurement and Performance Analysis of HPMA

Magnetic Field Measurement Platform and Measuring Method

In order to verify the analytical model, a measurement platform should be set up as Fig.2 shown to measure the magnetic field of HPMA.

HPMA stage under investigation, whose effective magnetic area is 212 mm × 212 mm and magnetic pole pitch τ_n=10.6 mm, is mounted on a specially designed x-y plane motion mechanism driven by two step motors so that HPMA stage can be controlled to motion in x- and y-direction, i.e. x-y plane. The Hall probe is installed on the measurement and adjustment mechanism, thus its Z-direction position can be adjusted with an accuracy of 0.1 mm. Teslameter (YL1020, Manufatured by Yiliang Sicence and Technology Co. Ltd, Beijing, China) initially processes the data Hall probe measured and transmits these data to a computer via serial communication.

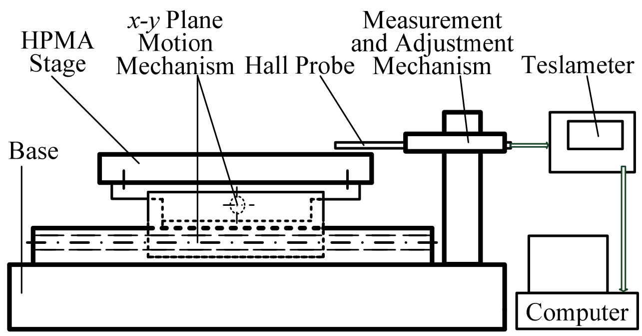

FIG. 2 MAGNETIC FIELD MEASUREMENT PLATFORM

Measurement Results and Performance Analysis of Magnetic Field

In x-y plane whose area is 190.8 mm × 190.8 mm, X component and Y component of MFDD are measured at different air gap Z ranging from 1.0 mm to 5.0 mm.

Fig.3 shows the measured results of X component of MFDD at air gap Z = 2.6mm, where (a) is the global view, (b) is the X-direction view and (c) is the Y-direction view. Other graphes can be drawn in accordance with other data measured in different components and air gaps. From these graphes, performances of magnetic field of HPMA can be concluded as follows:

- X components in X-direction and Y

components in *Y*-direction of MFDD are both dominated by sinusoidal distribution. The sine period is consistent with the structure of HPMA. This period equals double pole pitch of HPMA, i.e. $2\tau_n = 21.2$ mm.

- *X* components in *Y*-direction and *X* components in *Y*-direction of MFDD are basically constant. Yet there is a little amplitude pulse.
- *X* and *Y* components of MFDD both decays in *Z*-direction by natural exponential distribution. This result can be observed and analyzed from Fig. 5, Fig. 6.

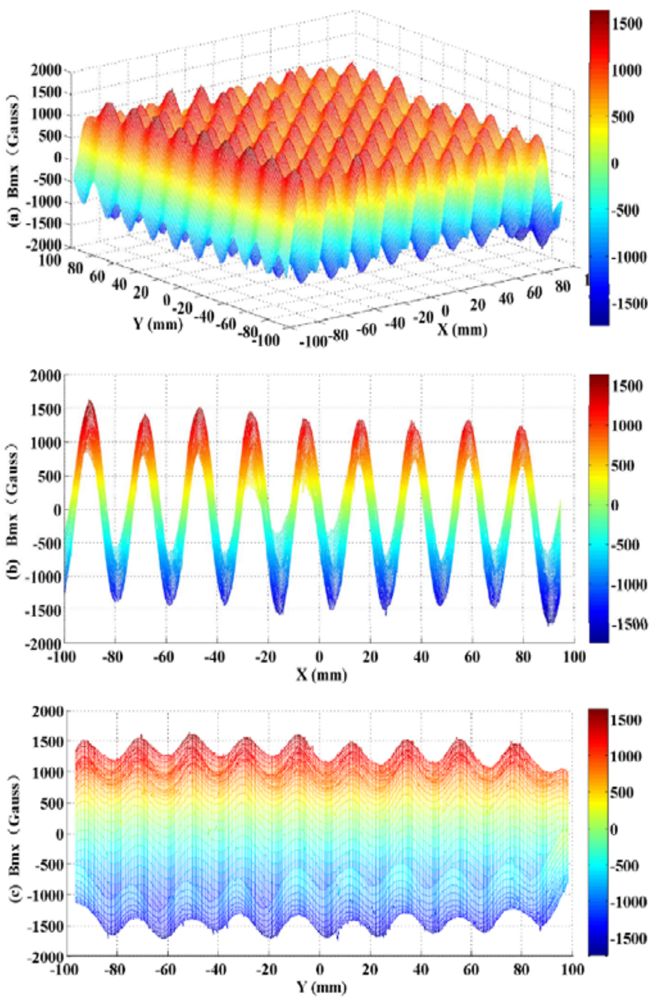

FIG. 3 MEARURED RESULTS OF X COMPONENT OF MFDD AT AIR GAP Z = 2.6MM

(A) GLOBAL VIEW (B) X-DIRECTION VIEW (C) Y-DIRECTION VIEW

These performances accords with the analytical model what Equations (2) represent, except that there exists in the end effect of magnetic field and a little amplitude pulse in measured results.

The area of HPMA under measurement is limited apparently and this reason can be explained by the end effect.

The amplitude pulse may result from two factors, one of which is the measurement system and the other is the higher order harmonic components of MFDD.

In order to judge which factor causes the amplitude pulse, the frequency and power spectrum of measured data of MFDD are analyzed. Fig. 4 displays the analyzed result of *Y* components of MFDD in *X*–direction. Frequency spectrum is derived from 256 points Discrete Fourier Transform (DFT) and power spectrum derived from Burg algorithm based on the 6th order AR model.

As it can be seen from Fig.4 that the amplitudes of frequency and power spectrum signals at 0.47 Hz, which is the 1st order harmonic of MFDD, are both more than 0 dB. In addition, the amplitudes of frequency and power spectrum signals whose frequency is more than 1.41 Hz, which denotes the higher order harmonic of MFDD, are not more than 0 dB.

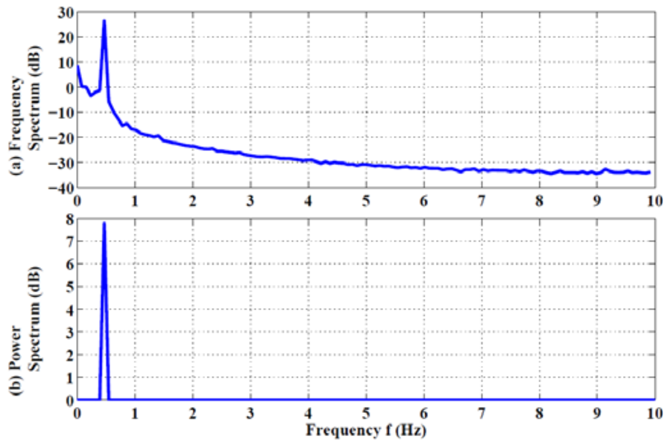

FIG. 4 FREQUENCY AND POWER SPECTRUM OF Y COMPONENT OF MFDD IN X-DIRECTION

Above results indicate that the 1st order harmonic component of MFDD is the dominant component and the other higher order harmonic component of MFDD can be neglected.

Key Parameter Fitting and Error Evaluation of Magnetic Field Model

Key Parameter Fitting

As it is shown in Fig. 3 that there are both a little amplitude pulse for *X* component in *X*–direction and *Y* component in *Y*-direction. Now that the dominant component of MFDD is the 1st order harmonic but not the other higher order harmonic components, the measurement system can be inferred to be the origin

of amplitude pulse. This is because there are inevitable installation and systematic errors for measurement system. So the measured data are processed by an averaging filter that can eliminate the measurement error

Fig. 5 and Fig.6 show the filtered *X* components in *X*–direction and the filtered *Y* components in *Y*- direction in different air gaps, respectively. From these two figures, the sine distribution of *X* component in *X*-direction and *Y* component in *Y*-direction can be observed clearly on one hand, on the other hand, the natural exponential decay of them in *Z*-direction can also be verified.

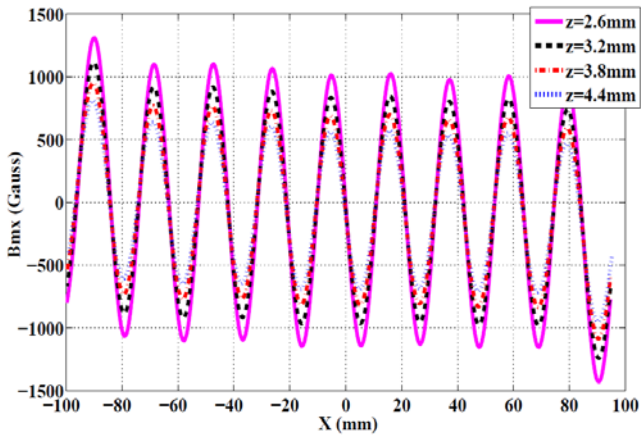

FIG. 5 FILTERED X COMPONENTS OF MFDD IN X-DIRECTION

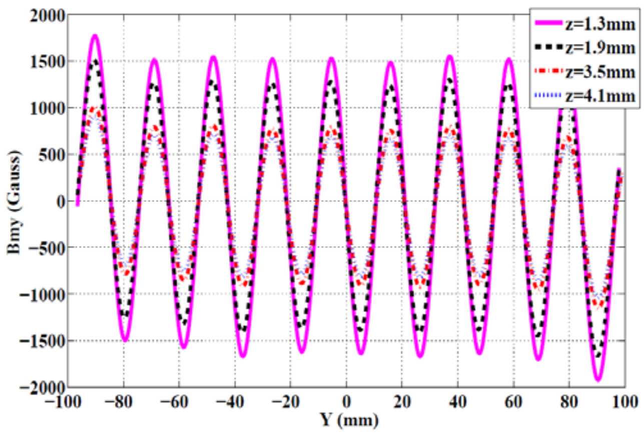

FIG. 6 FILTERED Y COMPONENTS OF MFDD IN Y-DIRECTION

For the analytical model of HPMA magnetic field Equations (2) represents, the key parameter B_{xy} is dependent upon the size and remanent magnetization of magnets. By fitting of different measured data in different air gaps, this parameter can be determined. For investgated HPMA whose magnetic pole pitch τ_n equals 10.6 mm and nominal remanent magnetization is 1.24 Tesla, the key parameter B_{xy} is fitted to be 6026 Gauss. So far, the analytical model of magnetic field of HPMA and its parameters are determined completely.

Error Evaluation of Magnetic Field Model

Based on the observation on the sine distribution of MFDD, either the analytical model or the measured results, the error root of mean square E_{rms} between the model value and the measured value is defined:

$$E_{rms}=\sqrt{\left[\sum_{i=1}^{n}(V_{mi}-V_{ti})^2\right]\Big/n}\ , \qquad (3)$$

where V_{mi} is the model value, V_{ti} is the measured value, and *i* is the measured points.

Table 1 shows the error root of mean square between the model value and the measured value for *X* component in *X*–direction and *Y* component in *Y*-direction in different air gaps. The measured points are 382 both for *X* component and *Y* component at the same air gap. i.e. $n = 382$ in Equation (3).

TABLE 1 THE ERROR ROOT OF MEAN SQUARE BETWEEN MODEL VALUE AND MEASURED VALUE FOR MFDD IN DIFFERENT AIR GAPS

Error Root of Mean Square Between Model Value and Measured Value for MFDD in Different Air Gaps			
Air gap (mm)	***X* Component (Gauss)**	**Air gap (mm)**	***Y* Component (Gauss)**
2.6	35.7779	1.3	43.5540
2.8	35.5131	1.5	41.3788
3.0	34.4283	1.7	40.4144
3.2	33.5806	1.9	39.2344
3.4	33.0977	2.1	38.4831
3.6	32.5934	2.3	37.8051
3.8	31.5512	3.5	35.1241
4.0	31.0960	3.7	33.6929
4.2	30.7509	3.9	32.5009
4.4	30.5123	4.1	31.1415

As it can be seen from Table 1 that the error root of mean square of 382 points are all less than 43.554 Gauss in different air gaps from 1.3 mm to 4.4 mm. If the relative error is defined as the ratio of the error root of mean square to the amplitude of sinusoidal MFDD, the maximum of these relative errors, of which the air gap is 4.4 mm, is 30.5123/610 = 5%. Thus, these relative errors decay with the air gap's decrease.

Conclusions

The present paper has faced the modeling problem of magnetic field of Halbach permanent magnet array.

The analytical first order harmonic model of magnetic flux density distribution for Halbach permanent

magnet array is initially obtained by neglecting all the higher order harmonic of Fourier harmonic waves and then transforming the coordinate system to the desired coordinate system.

The real magnetic field of a Halbach permanent magnet array is measured on *x*-*y* planes in different air gaps. Magnetic field performances are analyzed which accords with the measured results and they accords to the initially constructed magnetic field model.

The key parameter of the magnetic field model is fitted by the measured data. The error between the model value and the measured value of magnetic field is evaluated.

The analytical first order harmonic model of magnetic flux density distribution for Halbach permanent magnet array established completely can be used to model the electromagnetic force/torque and then to commutate the magnetically levitated planar motor.

ACKNOWLEDGMENT

This work was funded by the Introduction Talent Research Project of Northwest University for Nationalities (XBMUYJRC201301).

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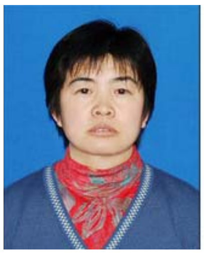

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